AUTHOR’S PREFACE

to

THE FIVE STAGES OF MATH ACHIEVEMENT

by

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Stage 1: Understanding Concepts Mathematically

Stage 2: Recalling and Assembly

Stage 3: Creating Frameworks for Problem-Solving

Stage 4: Developing & Using Algorithms for Problem-Solving

Stage 5: Exam Readiness Across Topics and Levels

AUTHOR'S PREFACE

The Stages proposed in this book have grown out of my own clinical experience of teaching Math to K-12 students or their equivalent in different parts of the world for over 35 years. A lot of it has been gleaned from clinical observations, teaching sessions, notes from clinical assessments, parent, student interviews, and reflections following my training workshops. Some aspects of the Stages have been distilled during my 15-year phase of developing the Karismath program.

The perspectives that I have drawn of the Five Stages are entirely mine, and open to debate.

The Five Stages of Math Achievement are being offered with certain qualifications and within a certain context:

1. The Five Stages cover the K-12 school range and suggest increasing stages of complexity in the understanding and applying of Math concepts for that range.

2. The Stages do not necessarily begin at any particular age. Stage 1 applies to students of all ages between K1-12, since everyone, at some point or another, is introduced to new mathematical concepts.

3. These Five Stages are unique and distinct in and by themselves, and are distributed across the progression of increased mathematical understanding. They can be clearly demarcated for assessment purposes.

4. More than one stage may co-exist during a particular period of math learning. Multiple stages may also be triggered off like a cascade during which different stages may get activated with varying gaps of separation between them.

5. The Stages also suggest to an extent, levels of cognitive readiness for assimilating increasing levels of mathematical complexity within the K-12 range.

In the construction of the Five Stages, I confess my bias against rootless learning. I believe that whatever is learned is tied to the learner's inexhaustible desire for meaning-making. The Stages have been presented also from the Math teaching point of view. My own experiences confirm that Math teaching principles and approaches bear fruit when they are in sync with the Five Stages. They also offer to an extent a framework for developing more useful evaluation methods.

These Stages are being proposed because there is an urgent need for Math teachers to ground their practice in some kind of a basic theory amidst a storm of conflicting and confusing arguments about the best way forward. The theory can be extended with more research and contribution from peers. The Five Stages, like the Karismath program, serve as the first small step in a journey of a thousand miles.

There is also a need to add an important dimension to the continuing debate on differentiated instruction. Such an instructional approach takes into account the learning habits, attitudes, interests, prior knowledge, personal preferences of learners. Current thinking suggests that success in offering differentiated education depends upon the effectiveness of far-sighted futuristic educational programs. This is true. But the expectations that drive such thinking may be misplaced. Many assume that an educational program should be designed to accommodate the varying needs of an infinite variety of learning styles, habits, prior knowledge, interests etc. Sadly, this may not be possible for a long time to come.

However, what new-generation educational programs in Math can do today is pedagogy-embedding i.e. design and create teaching and learning tools that actually *do* the Math teaching with minimal teacher-parent intervention. Such a program frees the most important ally to do the most needed task: personalize and customize Math education for differentiated learning. Learners recognize and experience the personalization and customization of learning environments when they feel better understood by their closest allies. Parents and teachers should analyze, diagnose and remedy their student's difficulties and errors in ways that are congruent with their student's felt needs. It is a good way to make them feel better understood and cared for. In some ways it serves the most important goals of differentiated learning. It could be that new generation Math programs will displace the traditional teacher's role quite radically: while the program does the teaching, the teacher does all the differentiating in learning.

Stages 1 and 2 begin with the slow transformation of Math concepts into functioning knowledge. At the beginning, mathematical concepts and their symbolic representations are assimilated in small, discrete and incremental doses. Between the first two stages they are recycled into bits and bytes of "mathematical knowledge".

From the teaching point of view, everything that is understood and learned during the Stages 1 and 2 should be experienced as part of a continuing Math Story, a long saga. Each step of the way learners receive bits and pieces that are not scattered fragments but complete units in themselves, like coherent chapters in a continuing saga. Like chapters in any story, they eventually end but the Math story continues. If teachers and parents, with the help of a Math program, succeed in generating their learners' interest and curiosity in the Math Story, as in an epic saga, then something of great value has been achieved.

Stages 1 and 2 assume that learners will be left with questions. Again, as in an epic saga, getting bits and pieces of a larger story, can be frustrating. But frustration often feeds on unsatisfied curiosity. Teachers should not hesitate to lead their students to wonder: "What happens next?". Even when learners may not *like* the Math they are doing they at least feel secure and satisfied about *understanding* what they learn, every step of the way. One can therefore rely on the exasperating curiosity that follows. "OK" they might say, " I GET this......but so what?"

Interpret this to mean: "What is all this leading to?". That's a good enough place to be. The Thousand and One Nights' tales began with the first story.

The Stages also take into account the existence of new-generation Math programs (like Karismath) that have pedagogy-embedded teaching and learning tools. These do the teaching directly. As a result, teachers and parents are freed from playing the conventional teacher role which requires their primary commitment to their own subject i.e. Math. In their new liberated role parents and teachers can appreciate the value of "teaching the learner" rather than "teaching

Math". When a program does much of the teaching, parents and teachers can afford the time to swing over to the side of the learner and observe him/her learn. It is a dramatic role-shift that can be felt immediately: from traditional teacher to a key ally in supporting a student's Math learning experiences.

Playing such a role helps to shed more light upon the learner's personality: his interests, her learning attributes, his level of understanding, her attitudes and so on. More importantly, teachers are better able to monitor student progress and understand *how* they learn. Five different learners can make the same mistake in Math for five different reasons. Often the reasons have to do with the way each of the five think of and respond to a given problem.

Knowing the learner helps parents and teachers correctly diagnose their difficulties, especially in the higher problem-solving Stages 4 and 5. The risks of attributing incorrect reasons for why a student performed poorly in a Test are otherwise great. Sometimes they are the main reason why students do not learn from their mistakes. The problem originates in their teachers doing the same. Many teachers and parents trust their personal diagnosis because they feel strongly about "incorrect" mathematical processes and the solutions that follow. However, correct and accurate diagnosis comes from an understanding and appreciation of the pattern of a learner's difficulties as well as the pattern of his/her mathematical errors.

The consequences of an incorrect diagnosis of errors and difficulties also contaminate teacher-learner relationships. When teachers stubbornly pursue incorrect remedies and prescriptions for remediation it gets harder to manage the growing alienation between them and learners. Students always know when a teacher is wrong. Not surprisingly, teachers and parents are often wrong about things that matter more to the students than Math. Which is why knowing the learner brings far greater benefits to teachers and parents than just "knowing their Math".

Needless to state, the opposite combination also comes with precarious risks: knowing your student but not knowing your Math.

Stages 3 and 4 both relate to problem-solving. There is a lot of emphasis on examining, understanding and interpreting word-problems rather than on "finding the right answers". This approach enables learners to become more process-oriented and less results-obsessed. It is an important digression from conventional approaches. Today's knowledge-economy demands a new skills-set: innovative ideas that lead to the creation and design of habit-transforming products in everyday lives. Often the solution already exists in terms of a "vision" for a new product. The skills-set needed for tomorrow will draw on the ability of knowledge-workers to examine how the procedural aspects of conception, creation, design and production can be managed in the making of desirable products (or known solutions). These must suit the needs of a rapidly changing world. The swift global spread of technology promises diverse delivery-platforms for goods and services. Many are now designed and produced in virtual environments. Already the trajectory for academic life and a successful career path is getting established in the screen-oriented habits of children and school goers as young as 5 years old. These are here to stay. Consequently Math education needs to adapt to a new social and global environmental and prepare to redefine learning outcomes for this century.

Some of the most pressing questions around Math education are addressed in Stage 5. These are provoked by the observation that Stage 4 learners, despite attaining a high level of mastery in problem-solving, continue to perform below expectations in major examinations. This is because final examinations, or state wide, provincial or national examinations cover multiple Math Topics. They demand a wider range of recall and application of math concepts and algorithms, from students. Nowhere is failure due to insufficient examination-readiness more evident than in Stage 5. And yet, everywhere in the world, bottom-line student proficiency in high school Math is evaluated via such broad-spectrum examinations covering multiple Math Topics. A small minority performs well. Most succumb to the psychological pressures of time, anxiety and an attack of nerves. Some of it can be attributed, of course, to lack of preparedness. But other more serious reasons exist beneath the surface.

Viewed from the macro perspective many questions arise as to why a majority of the world's student population fails Math. What is it that most nations share in common with each other when it comes to the systemic delivery of Math education? What is so unique about the history of Math education that anticipates the failure of an overwhelming majority as perfectly normal and understandable? Why does public opinion unflappably support the belief that Math is tough and only smart kids can excel in it?

When viewed from the micro perspective, other issues come to surface. Most are related to the design and content of Math curriculums everywhere. One could argue that there is something in the design of the curriculum that makes exam preparedness at the level demanded, a difficult, if not impossible proposition for most students. For one thing, the school system does not provide enough time or resources within the math curriculum. How much review and rehearsal work is required for attaining 100% examination preparedness across multiple Topics covered over, say, the last 2 years of school ? A lot. Compared to the time and resources currently allocated to schools, the amount actually needed would be dismissed as being outrageously phenomenal. How then, does one explain the continuing success of a minority group functioning in Stage 5 despite these limitations? What can we learn from them? More importantly, what in their practices, are not replicable or reliable enough to help the vast majorities of failing students ? Where then do we turn for solutions?

Admittedly, in the book such concerns are raised within the context of the Five Stages of Math Achievement. They give rise to many more questions that fall outside the purview of my discourse. Some suggestions are offered from the curricular perspective, while challenging the urban myth: “Math is for Smart Kids Only”.

Indeed there exist other urban myths around Math. Some aren’t myths but just misconceived notions that have simply gone unchallenged. For instance, a continuing but stubbornly ignored question regarding Math curriculums the world over is : “Who needs all the Math taught in schools, in *real* life?” Successful professionals from all walks of life (including Science!) have repeatedly protested that of all the Math they learned at school, only 1-3% of it is retained for actual use in their everyday working lives. The rest is doomed to pure redundancy. Hence, why the need to impose a wider Math curriculum upon the entire school-going population? Perhaps much could be gained from the time and resources saved by minimizing the width and depth of the Math curriculum across K-12. The savings could, perhaps, be diverted to delivering a high quality, though minimalist Math curriculum that attracts far greater responsiveness from students. Arguably, such a curriculum could inspire and motivate students to attain much higher levels of success in Math than is presently possible, or even conceivable.

The Five Stages does not address such issues or others related to them. But it may make questions such as the one above, more understandable, possibly even more compelling and urgent. More so, as long as:

(i) Mathematics continues to be touted in narrow medieval terms as the "Language of Science" and little else.
(ii) Mathematics is perceived as something "inherently complex, difficult and accessible only to superior minds."
(iii) The K-12 Math curriculums across the world remains unchanged in its rickety design and content.
(iv) Math pedagogy is not synchronized with current research on how evolving minds understand quantity, numeracy, numerical reasoning and how they process numerical information.
(v) Math teaching programs do not create pedagogy-embedded worksheets whose design guides the learning process

Very few cogent reasons are offered to parents, teachers and the general public re: the rationale behind existing Math curriculums. Let us accept for gospel truth that mediocre or inferior Math achievement is the established scientific norm for a majority. The bell-curve confirms that some will do poorly in Math, a majority will receive average grades in Math, and a few will excel in math. That goes for running or tying shoe-laces. So what is this fuss all about? That's not so difficult to answer since everyone knows what the fuss is really about. It orbits around the following fact:

"*The United States has one of the worst high school dropout rates in the industrialized world, and its students regularly rank far below those in other Western and Asian countries in reading and math scores. Slightly more than half of the population has only a high school diploma. One out of every two American university students drops out before completing their post-secondary studies*."

Secondly, "average" grades (determined decades ago) in Traditional Math curriculums for a 21st century population of Math learners does not necessarily translate into anything meaningful or revealing. It could merely suggest that the probability of the curriculum failing our students may be the same as for our students failing the curriculum.

Thirdly, the US has, to this day, displayed global leadership in higher education. It has the highest number of Ph.D's, high-tech engineers and technicians, researcher faculties and members, think tanks and universities in relation to its population. It has also the highest number of patent ownerships and pending patents in the world (recent competition, and a serious one in patent ownership is now coming from China and India). This is the outstanding record of a nation that has understandably led the world in seeding the knowledge-economy that is harvested by many nations. How can one then accept the dismal record of mediocrity and failure in US schools? It is not just a matter of national pride. True, many in the US are distressed by school failure in Math because it is a sad betrayal of the standards that the US has set for the world in most academic disciplines. Many more worry about what continuing failure portends for the US national economy in the coming decades.

The world at large, including North America, may justify the existence of the age-old "national Math curriculums" on moral grounds. This may surprise many but here is how the argument goes:

"We have to offer Math, with its broad curricular sweep from K-12, to all students because it is their inherent right. We cannot offer it to brighter students because that would be discriminatory. By following such an equitable approach, we cannot be blamed for not having offered each and every student an equal opportunity to succeed and become an aeronautical engineer, or an astronaut, pilot, engineer, scientist, physicist, doctor, and so on. The burden of failure and its consequences fall upon their shoulders, not ours"

The most expensive Hotel in London takes mischievous pride in its claim that: "The revolving doors of our Hotel are open to all without any discrimination whatsoever".

The analogy holds for the offer of Math education for all. There remain systemic pitfalls, gender-bias hurdles, curricular limitations, inadequate design, social and pedagogical inequities plaguing the delivery of “Math education for all”. When all of these are set into motion, only the privileged few get to pass through its many revolving doors. It is odd that all those doors are powered by the notion of democratic equality.

The Five Stages of Math, as proposed in this book, are not set in stone. They do not answer many questions, nor do they raise others that readers are likely to. The Stages will certainly undergo some transformations in their form and content. It is hoped that valuable contributions from my peers will hone the Stages further with sharper edges. I expect that its evolution into a more elegant and comprehensive form will continue with extensive support from formal research.

In the meantime I hope that readers accept the Stages in the spirit in which it is offered: to provoke intelligent debate and discourse on all those thoughts, ideas and feelings that we hold very dear when it comes to Math education for our children.

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